Quantitative Runge type approximation theorems for kernels of partial differential operators

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Abstract:
A well known consequence of Runge's classical theorem on rational approximation is that for open subsets $X_1 \subseteq X_2$ of the complex plane $\mathbb{C}$, every holomorphic function on X_1 can be approximated by a holomorphic function on X_2 with respect to the compact-open topology, precisely when X_2 does not contain a compact component of $\mathbb{C}\backslash X_1$. In the 1950s, this approximation theorem has been generalized independently by Lax [4] and Malgrange [5] from holomorphic functions, i.e. functions in the kernel of the Cauchy-Riemann operator, to kernels of elliptic constant coefficient differential operators.

Recently, there has been an renewed interest in such approximation results, namely in so-called quantitative Runge type approximation results for elliptic operators of second order on the one hand (cf. [6]) and generalizations of the Lax-Malgrange Theorem to certain classes of non-elliptic operators on the other hand (cf. [1], [2], [3]).

In our talk, we show how recent results on Vogt's and Wagner's topological invariant (Ω), introduced in [7], for kernels of partial differential operators can be used to obtain quantitative Runge type approximation results for elliptic operators of arbitrary order, non-degenerate parabolic operators and the one dimensional wave operator.

The talk is based on joint work with A. Debrouwere (University of Brussels).

References

[1] A. Enciso, M.A. García-Ferrero, D. Peralta-Salas, *Approximation theorems for parabolic equations and movement of local hot spots.* Duke Math. J. 168(5):897–939, 2019.

[2] A. Enciso, D. Peralta-Salas, *Approximation theorems for the Schrödinger equation and quantum vortex reconnection.* Comm. Math. Phys. 387(2):1111–1149, 2021.

[3] T. Kalmes, *An approximation theorem of Runge type for certain non-elliptic partial differential operators.* Bull. Sci. Math. 170, Paper No. 103012, 26 pp. 2021.

[4] P. D. Lax, *A stability theorem for solutions of abstract differential equations, and its application to the study of the local behavior of solutions of elliptic equations.* Comm. Pure Appl. Math. 9:747–766, 1956.

[5] B. Malgrange, *Existence et approximation des solution des équations aux derivées partielles et des équation de convolution.* Ann. Inst. Fourier (Grenoble) 6:71–355, 1955-56.

[6] A. Rüland, M. Salo, *Quantitative Runge approximation and inverse problems.* Int. Math. Res. Not. IMRN 20:6216–6234, 2019.

[7] D. Vogt, M.J. Wagner, *Charakterisierung der Quotientenräume von s und eine Vermutung von Martineau.* Studia Math. 67(3):225–240, 1980.