

NONLINEAR CONDITIONS FOR ULTRADIFFERENTIABILITY

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A remarkable theorem of Joris states that a function $f : \mathbb{R}^d \rightarrow \mathbb{C}$ is of class C^∞ if two coprime powers of f are of class C^∞ . Thilliez showed that this result carries over to suitable Denjoy–Carleman classes of Roumieu type, where $d = 1$. Refining Thilliez’ methods, I will show that the theorem of Joris is valid for all ultradifferentiable classes \mathcal{C} with basic stability properties (including Braun–Meise–Taylor classes and classes defined by weight matrices of Beurling and Roumieu type), in all dimensions (even on infinite dimensional Banach spaces and convenient vector spaces), no matter if quasianalytic or not. The core of the proof is based on uniform unidirectional holomorphic approximation and almost analytic extension.

With this result at disposal, we then derive a full characterization of the analytic germs $\Phi : (\mathbb{K}, 0) \rightarrow (\mathbb{K}^n, 0)$ (where \mathbb{K} is \mathbb{R} or \mathbb{C}) such that $\Phi \circ f \in \mathcal{C}$ implies $f \in \mathcal{C}$, for all continuous function germs $f : (\mathbb{R}^d, 0) \rightarrow (\mathbb{K}, 0)$, in terms of a condition on the support of the Taylor series of Φ .

(Joint work with David Nenning and Gerhard Schindl)