

Generalized interpolation: a functorial point of view

part 1

Thomas Lamby and Samuel Nicolay

Many spaces, such as the Besov spaces $B_{p,q}^s$, naturally arise through the (real) interpolation theory: $B_{p,q}^s = [H_p^t, H_p^u]_{\alpha,q}$ roughly means that this Besov space “lies” between the Sobolev spaces H_p^t and H_p^u , with $s = (1 - \alpha)t + \alpha u$. A classical generalization of these interpolation methods has been introduced in the eighties. The idea is to replace the linear formula $(1 - \alpha)t + \alpha u$ by a more general function $\alpha \mapsto f(\alpha)$. As the interpolation theory can be expressed using the language of categories, it is natural to do the same with the generalized interpolation spaces. This is the purpose of these talks.

We first discuss the existing relations between the Boyd functions and the admissible sequences, with a particular interest to the Boyd indices. These notions are intended to be tools in order to generalize some functional spaces. We then define interpolation functors depending on Boyd functions from the category of compatible normed vector spaces to the category of normed vector spaces. Next, we generalize some classic results of interpolation theory and apply them to some classical functional spaces.

This talk is divided into two parts. The first talk will be more about the general notions.