CONTINUOUSLY DIFFERENTIABLE FUNCTIONS ON COMPACT SETS

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Abstract. We consider the space $C^1(K)$ of $\mathbb{R}^m$-valued continuously differentiable functions $f : K \to \mathbb{R}^m$ on an arbitrary compact set $K \subseteq \mathbb{R}^d$ defined by the usual affine-linear appraximability, i.e., there is a continuous function $f' : K \to L(\mathbb{R}^d, \mathbb{R}^m)$ with

$$\frac{\|f(x+h) - f(x) - f'(x)(h)\|_{\mathbb{R}^m}}{\|h\|_{\mathbb{R}^d}} \to 0 \text{ for all } x \in K$$

endowed with the norm $\|f\|_{C^1(K)} = \inf\{\sup \|f'(x)\|_{L(\mathbb{R}^d,\mathbb{R}^m)} : x \in K\} :$ f' is a continuous derivative of $f\}$.

We characterize the completeness of this space and prove that the restriction space $C^1(\mathbb{R}^d|K) = \{f|_K : f \in C^1(\mathbb{R}^d)\}$ is always dense in $C^1(K)$. The space $C^1(K)$ is then compared with other spaces of differentiable functions on compact sets.

This is a joint work with Leonhard Frerick and Laurent Loosveldt.

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