Global regularity in ultradifferentiable spaces for non hypoelliptic PDE

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The problem of global regularity for partial differential operators was first introduced by Shubin in the frame of Schwartz functions and tempered distributions; a linear operator $A : \mathcal{S}' \to \mathcal{S}'$ is said to be regular if the conditions $u \in \mathcal{S}'$, $Au \in \mathcal{S}$ imply that $u \in \mathcal{S}$. Shubin formulates an hypoellipticity condition (in his global pseudodifferential calculus), that is sufficient to have regularity of the correponding operator. On the other hand, such hypoellipticity is far to be necessary, as there are several examples of operators which are not hypoelliptic but are globally regular (such as the Twisted Laplacian). The problem of characterizing global regularity for classes of operators is quite hard. Even in very particular cases (as for ordinary differential operators with polynomial coefficients) necessary and sufficient conditions for global regularity are not known.

In this talk we present some results on global regularity of (non hypoelliptic) linear partial differential operators with polynomial coefficients in non isotropic ultradifferentiable classes of Beurling type, using techniques from time-frequency analysis; the techniques are related to the transformations of the operator itself by a quadratic representation of Wigner type.

This talk is based on collaborations with **E. Buzano, C. Boiti, D. Jornet, C. Mele**, cf. [1, 2, 3].

References

[1] C. Boiti, D. Jornet, A. Oliaro, *Regularity of partial differential operators in ultradifferentiable spaces and Wigner type transforms*, J. Math. Anal. Appl. 446 (2017), 920-944.

[2] E. Buzano, A. Oliaro, *Global regularity of second order twisted differential operators*, J. Differential Equations 268, 7364–7416 (2020).

[3] C. Mele, A. Oliaro, *Regularity of global solutions of partial differential equations in non isotropic ultradifferentiable spaces via time-frequency methods*, J. Differential Equations 286, 821–855 (2021).