Some results on the Bloch space over the unit ball of a Hilbert space

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Based on joint works with

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Definition

An analytic function $f : \mathbb{D} \to \mathbb{C}$ is said to be a *Bloch* function if

$$\|f\|_{\mathcal{B}} := \sup_{|z|<1} \{(1 - |z|^2)|f'(z)|\} < +\infty.$$

The space of all such functions, denoted by $\mathcal{B}$, is the Bloch space of $\mathbb{D}$.

Endowed with the norm $|f'(0)| + \|f\|_{\mathcal{B}}$, $\mathcal{B}$ becomes a Banach space.

For every automorphism $\varphi_a(z) = \frac{z-a}{1-\bar{a}z}$ of $\mathbb{D}$ it turns out that $\|f \circ \varphi_a\|_{\mathcal{B}} = \|f\|_{\mathcal{B}}$. That is, $\mathcal{B}$ is invariant under automorphisms. Actually, it is the maximal automorphism invariant analytic function space.

Further information on the Bloch space as well as some historic details may be seen in Zhu's book [8].

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R. M. Timoney extended Bloch functions to bounded homogeneous domains $\mathcal{D}$ of $\mathbb{C}^n$ (see [5] and [6]). And that can be done in several ways:

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The space of all such functions, denoted by $\mathcal{B}(B_n)$ is the Bloch space of B_n.

Realize that here $f'(z)$ is given by the gradient vector $\nabla f(z) := (\frac{\partial f}{\partial z_1}(z), \cdots, \frac{\partial f}{\partial z_n}(z))$. So $\|f'(z)\| = \|\nabla f(z)\|$.

Timoney also considered analytic functions satisfying

$$\sup_{z \in B_n} (1 - \|z\|^2)|Rf(z)| < \infty,$$

where $Rf(z) := \langle \nabla f(z), \bar{z} \rangle$ is the so-called radial derivative of f in z, here $\bar{z} = (\bar{z_1}, \ldots, \bar{z_n})$.

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Adding up $|f(0)|$ to each of the quantities above to avoid constant functions to have norm 0, we get Banach spaces. It was shown by Timoney that such are equivalent norms in the space $\mathcal{B}(B_n)$.

However, these definitions using the gradient or the radial derivative, do not allow us to define a semi-norm invariant by automorphisms. It seems that this is why Timoney used the Bergman metric on B_n to define the norm $Q_f(z)$ of df as a cotangent vector, so f is said to belong to the Bloch functions space on B_n if $\|f\|_{\mathcal{B}(B_n)} := \sup_{z\in B_n} Q_f(z) < \infty$. This expression satisfies $\|f \circ \varphi\|_{\mathcal{B}(B_n)} = \|f\|_{\mathcal{B}(B_n)}$ for any $\varphi \in Aut(B_n)$ since the Bergman metric is also invariant by automorphisms. Timoney also proved that this is equivalent to the previous formulations and got the bonus of the invariance under automorphisms.

Recall that the invariant gradient of a holomorphic function $f : B_n \to \mathbb{C}$ at $z \in B_n$ is $\widetilde{\nabla} f(z) := \nabla(f \circ \varphi_z)(0)$. Zhu proved that a holomorphic function $f : B_n \longrightarrow \mathbb{C}$ belongs to the Bloch space $\mathcal{B}(B_n)$ if and only if $\sup_{z\in B_n} \|\widetilde{\nabla} f(z)\| < \infty$ (see [7]).

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Let $(e_k)_{k\in\Gamma}$ be an orthonormal basis of E that we fix at once. Then every $z \in E$ can be written as $z = \sum_{k\in\Gamma} z_k e_k$ and we write $\overline{z} = \sum_{k\in\Gamma} \overline{z_k} e_k$.
Given a holomorphic function $f : B_E \to \mathbb{C}$ and $x \in B_E$, we will denote, as usual, by $\nabla f(x)$ the gradient of f at x, that is, the unique element in E representing the linear operator $f'(x) \in E^*$. It may be written $\nabla f(x) = \left(\frac{\partial f}{\partial x_k}(x)\right)_{k\in\Gamma}$, and so

$$f'(x)(z) = \sum_{k\in\Gamma} \frac{\partial f}{\partial x_k}(x) z_k = \langle z, \overline{\nabla f(x)} \rangle.$$

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$$\|f\|_{\mathcal{B}(B_E)} := \sup_{x \in B_E} (1 - \|x\|^2)\|\nabla f(x)\| < \infty.$$

As usual $|f(0)| + \|f\|_{\mathcal{B}(B_E)}$ is a complete norm on $\mathcal{B}(B_E)$.

Bearing in mind the classical Bloch spaces defined on the unit ball of $\mathbb{C}$ and $\mathbb{C}^n$, and their possible definitions, we set the following possible natural norms on the space for the unit ball of E. Descriptions in terms of the radial derivative as in the finite dimensional case can be obtained as well.

Definition

For a holomorphic function $f : B_E \to \mathbb{C}$ we set

$$\|f\|_{\mathcal{R}} := \sup_{x \in B_E} (1 - \|x\|^2)|\mathcal{R}f(x)|,$$

where $\mathcal{R}(f)(x) = \langle \nabla f(x), \bar{x} \rangle$, $x \in B_E$.

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We denote $\mathcal{B}_{\mathcal{R}}(B_E)$ the space of holomorphic functions on B_E for which $\|f\|_{\mathcal{R}} < \infty$. As usual, $|f(0)| + \|f\|_{\mathcal{R}}$ is a complete norm.

Bearing in mind the classical Bloch spaces defined on the unit ball of $\mathbb{C}$ and $\mathbb{C}^n$, and their possible definitions, we set the following possible natural norms on the space for the unit ball of E.

Proposition

The spaces $\mathcal{B}_{\mathcal{R}}(B_E)$ and $\mathcal{B}(B_E)$ coincide. Hence their norms are equivalent.

The analogues of Möbius transformations on E are the mappings $\varphi_a : B_E \longrightarrow B_E$, $a \in B_E$, defined according to

$$\varphi_a(x) = (s_a Q_a + P_a)(m_a(x)) \tag{3.1}$$

where $s_a = \sqrt{1 - \|a\|^2}$, $m_a : B_E \longrightarrow B_E$ is the analytic map

$$m_a(x) = \frac{a - x}{1 - \langle x, a \rangle}, \tag{3.2}$$

$P_a : E \longrightarrow E$ is the orthogonal projection along the one-dimensional subspace spanned by a and $Q_a : E \longrightarrow E$ is the orthogonal complement, $Q_a = Id - P_a$. The automorphisms of the unit ball B_E turn to be compositions of such analogous Möbius transformations with unitary transformations U of E, that is, linear self-maps of E satisfying $\langle U(x), U(y) \rangle = \langle x, y \rangle$ for all $x, y \in E$.

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Definition

Denote $\mathcal{B}_{inv}(B_E)$ the set of holomorphic functions $f : B_E \to \mathbb{C}$ such that

$$\|f\|_{inv} := \sup_{x \in B_E} \|\widetilde{\nabla} f(x)\| < \infty.$$

Again $|f(0)| + \|f\|_{inv}$ is a norm in $\mathcal{B}_{inv}(B_E)$.
It is clear that $\|f \circ \varphi\|_{inv} = \|f\|_{inv}$ for any $f \in \mathcal{B}(B_E)$ and any automorphism φ of B_E.

Theorem

Let $f : B_E \to \mathbb{C}$ be a holomorphic function. Then $f \in \mathcal{B}(B_E)$ if and only if $f \in \mathcal{B}_{inv}(B_E)$.

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$$H^\infty(B_E) := \{f : B_E \to \mathbb{C} : f \ \ \textit{holomorphic and bounded}\}$$

is a uniform Banach algebra when endowed with the sup-norm $\|f\|_\infty = \sup\{|f(x)| : x \in B_E\}$. It is, obviously, the analogue of the space H^∞ for E.

Corollary

The inclusion $i : H^\infty(B_E) \longrightarrow \mathcal{B}(B_E)$ is a linear operator satisfying

$$\|f\|_{\mathcal{B}(B_E)} \leq \|f\|_\infty.$$

Corollary

Let $f \in H^\infty(B_E)$ with $\|f\|_\infty = 1$ and $\varphi \in \mathcal{B}$. Then $g = \varphi \circ f \in \mathcal{B}(B_E)$ and $\|g\|_{\mathcal{B}(B_E)} \leq \|\varphi\|_{\mathcal{B}}$.
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From now on $(X, \|\cdot\|)$ denotes a semi-Banach space of analytic functions on the unit ball of a Hilbert space E that is *invariant* under automorphisms φ of the ball B_E in the sense that for all $f \in X$, we have $f \circ \varphi \in X$ and $\|f \circ \varphi\| = \|f\|$.

Theorem

Assume that there is in X a nonconstant function and that there is a nonzero linear functional L on X that is continuous for the compact open topology τ_0. Then $X \subset \mathcal{B}(B_E)$. If further $L(1) \neq 0$, then $X \subset H^\infty(B_E)$.

Lemma

(a) Every term in the Taylor series of $f \in X$, belongs to X as well.
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Sketch of the proof for the case $L(1) = 0$.

For every automorphism φ of B_E consider the linear functional $L_\varphi := L \circ C_\varphi$, on X.

Using Lemma above one proves that given $e \in E$, some $L_\varphi(e^*) \neq 0$. So suppose L itself satisfies $L(e^*) \neq 0$.

By the τ_0 continuity of L, there is a balanced compact subset of B_E such that $|L(f)| \leq A \sup_{z \in M} |f(z)|$. After some calculations , there is a constant $C > 0$ such that

$$|L(\nabla f(0)^*)| \leq C\|f\| \quad \forall f \in X. \tag{5.1}$$

And further that for a given orthonormal basis $\{e_j\}_{j \in J}$ in E, the linear form $(\alpha_j) \in E \rightsquigarrow \sum \alpha_j L(e_j^*)$ is a continuous one.

Set $\varpi := (\overline{L(e_j^*)}) \in E \setminus \{0\}$ because $L(e_1^*) \neq 0$ and put $v = \frac{\varpi}{\|\varpi\|}$. Thus, $L((\alpha_j)^*) = \sum \alpha_j L(e_j^*) = < (\alpha_j), \varpi >$.

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