Mean ergodic composition operators on spaces of holomorphic functions

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Introduction

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- A map $p : X \to Y$ is in $\mathcal{P}(^m X, Y)$ if there exists a continuous m-linear form $L : X \times \overset{m}{\cdots} \times X \to Y$ such that $L(x, \overset{m}{\dots}, x) = p(x)$ for all $x \in X$.

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- For any $p \in \mathcal{P}(^m X, Y)$, $\lambda \in \mathbb{C}$ and $x \in X$ we have:

$$p(\lambda x) = \lambda^m p(x)$$

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Let $B_X \subset X$ denote the open unit ball (or simply B). A function $f : B \to Y$ is holomorphic if for each $a \in B$ there are $p_m \in \mathcal{P}(^m X, Y)$ such that

$$f(x) = \sum_{m=0}^{\infty} p_m(x - a),$$

converges uniformly on some open ball $B(a, r) \subset B$.

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- We denote by $H_b(B)$ the space of functions on $H(B)$ which are of bounded type. Each $0 < r < 1$ defines a seminorm

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- We denote by $H^\infty(B)$ the space of functions on $H(B)$ which are of bounded in B. The norm is given by

$$\|f\| := \sup_{x \in B} |f(x)|.$$

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- φ is the **symbol** of the composition operator.
- We have $C_\varphi : H_b(B) \to H_b(B)$ is well defined if and only if for each $0 < r < 1$ there is $0 < s < 1$ such that

$$\varphi(rB) \subseteq sB.$$

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Dynamical properties:

- Power boundedness
- Topologizability
- Mean ergodicity
- Uniform mean ergodicity

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- $T^0 = Id$,
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- The n-th Cesàro mean

$$T_{[n]} := \frac{1}{n} \sum_{m=0}^{n-1} T^m.$$

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- **Power Bounded**: $(T^n)_n$ is equicontinuous in $\mathcal{L}(E)$.
- **Topologizable**: there exist $a_n > 0$ such that $(a_n \cdot T^n)_n$ is equicontinuous in $\mathcal{L}(E)$.

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- **Topologizable**: there exist $a_n > 0$ such that $(a_n \cdot T^n)_n$ is equicontinuous in $\mathcal{L}(E)$.
- **Mean Ergodic (ME)**: $(T_{[n]})_n$ converges in the topology of pointwise convergence of $\mathcal{L}(E)$ (strong operator topology when E is Banach).
- **Uniformly Mean Ergodic (UME)**: $(T_{[n]})_n$ converges in the topology of bounded convergence of $\mathcal{L}(E)$ (operator norm topology when E is Banach).

Introduction

Proposition (Bonet, Domański)

Let U be a connected domain of holomorphy in $\mathbb{C}^d$ and let $\varphi : U \to U$ a holomorphic mapping. T.F.A.E.:

a. $C_\varphi : H(U) \to H(U)$ *is power bounded.*
b. $C_\varphi : H(U) \to H(U)$ *is uniformly mean ergodic.*
c. $C_\varphi : H(U) \to H(U)$ *is mean ergodic.*
d. φ *has stable orbits.*

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c. $C_\varphi : H(U) \to H(U)$ *is mean ergodic.*
d. φ *has stable orbits.*

When X is infinite dimensional:

- $H(B)$ is a locally convex semi-Montel space (not barrelled).
- $H_b(B)$ is a Fréchet space (not Montel).
- $H^\infty(B)$ is a Banach space (not Montel).

Stable orbits vs B-stable orbits

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Remark

By Schwarz Lemma, if $\varphi(0) = 0$ we have for all $n \in \mathbb{N}$ and $x \in B$

$$\|\varphi^n(x)\| \leq \|x\|.$$

And φ has B-stable orbits.

Examples

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  - Has B-stable orbits
  - Has not stable orbits

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- $F(x_1, x_2, x_3, \dots) = F(0, x_1, x_2, \dots)$
  - Has B-stable orbits
  - Has not stable orbits
- $S(x_1, x_2, x_3, \dots) = S(x_2, x_3, x_4, \dots)$
  - Has B-stable orbits
  - Has stable orbits

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  - Has not stable orbits
- $S(x_1, x_2, x_3, \dots) = S(x_2, x_3, x_4, \dots)$
  - Has B-stable orbits
  - Has stable orbits
- $\phi(x_1, x_2, x_3, \cdots) = (\frac{x_1}{2} + \frac{1}{2}, 0, 0, \cdots)$
  - Has not B-stable orbits
  - Has not stable orbits

The Hilbert space case

Let H be a Hilbert space and denote B_H its open unit ball. For each $a \in B_H$ we can find a map $\alpha_a : B_H \to B_H$ such that...

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- It is an automorphism
- It is holomorphic
- For each $0 < r < 1$ there is $0 < s < 1$ such that $\alpha_a(rB) \subseteq sB$
- $\alpha_a(a) = 0$ and $\alpha_a(0) = a$
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Remark

Consider $\varphi : B_H \to B_H$ a holomorphic map such that $\varphi(a) = a$ for some $a \in B_H$. Then

$$(\alpha_a \circ \varphi \circ \alpha_a)(0) = 0.$$

And $\alpha_a \circ \varphi \circ \alpha_a$ has B_H-stable orbits. Consequently, φ has B_H-stable orbits.

$H(B)$: Power bounded

Theorem

Let $\varphi : B \to B$ be holomorphic. Then the following are equivalent:

- *φ has stable orbits.*
- *$C_\varphi : H(B) \to H(B)$ is power bounded.*
- *$\left(\frac{1}{n} C_\varphi^n\right)_n$ is equicontinuous in $\mathcal{L}(H(B))$.*
- *$C_\varphi : H(B) \to H(B)$ is topologizable.*

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(Mujica) Let $K \subset B$ be a compact subset. Then this set is compact

$$\widehat{K}_{H(B)} = \{x \in B : |f(x)| \leq \sup_{y \in K} |f(y)| \text{ for every } f \in H(B)\}$$

$H(B)$: Power bounded $\Rightarrow$ UME

Proposition (Bonet, de Pagter, Ricker)

Let E be a semi-Montel lcHs. Then every power bounded operator on E is uniformly mean ergodic.

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The space $H(B)$ is semi-Montel.

Corollary

Let $\varphi : B \to B$ be holomorphic mapping. If $C_\varphi : H(B) \to H(B)$ is power bounded, then it is uniformly mean ergodic.

$H_b(B)$: Power bounded

Theorem

Let $\varphi : B \to B$ be holomorphic of bounded type. Then the following are equivalent:

- φ *has* B*-stable orbits.*
- $C_\varphi : H_b(B) \to H_b(B)$ *is power bounded.*
- $\left(\frac{1}{n} C_\varphi^n\right)_n$ *is equicontinuous in* $\mathcal{L}(H_b(B))$*.*
- $C_\varphi : H_b(B) \to H_b(B)$ *is topologizable.*

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- *$C_\varphi : H_b(B) \to H_b(B)$ is topologizable.*

Fix $0 < r < 1$. For every subset $A \subseteq rB$ there is $0 < s < 1$ such that

$$\widehat{A}_{H_b(B)} = \{x \in B : |f(x)| \leq \sup_{y \in A} |f(y)| \text{ for every } f \in H_b(B)\}$$

is contained in sB.

$H_b(B)$: ME $\Rightarrow$ Power bounded

Remark

If $C_\varphi : H_b(B) \to H_b(B)$ is mean ergodic, the sequence $\left(\frac{1}{n} C_\varphi^n\right)_n$ converges to 0 pointwise. Then it is equicontinuous in $\mathcal{L}(H_b(B))$.

$H_b(B)$: ME $\Rightarrow$ Power bounded

Remark

If $C_\varphi : H_b(B) \to H_b(B)$ is mean ergodic, the sequence $\left(\frac{1}{n} C_\varphi^n\right)_n$ converges to 0 pointwise. Then it is equicontinuous in $\mathcal{L}(H_b(B))$.

Proposition

If $C_\varphi : H_b(B) \to H_b(B)$ is mean ergodic, then C_φ is power bounded.

$H_b(B)$: Power bounded $\not\Rightarrow$ ME

The following operators are power bounded but not mean ergodic

- $C_S : H_b(B_{c_0}) \to H_b(B_{c_0})$
- $C_F : H_b(B_{\ell_1}) \to H_b(B_{\ell_1})$

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Idea: The space $H_b(B_X)$ contains a complemented copy of X'.
We have $C_S|_{\ell_1} = F : \ell_1 \to \ell_1$ and $C_F|_{\ell_\infty} = S : \ell_\infty \to \ell_\infty$ are not mean ergodic.

$H_b(B)$: Sometimes... Power bounded $\Leftrightarrow$ ME

We say that function $p : X \to \mathbb{C}$ is in $\mathcal{P}(X)$ (is a polynomial) if for some $M \in \mathbb{N}_0$ we have

$$p = \sum_{m=0}^{M} p_m,$$

where $p_m \in \mathcal{P}(^m X)$ for $m > 0$ and $p_0 : X \to \mathbb{C}$ is a constant function.

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Proposition

Assume that $\varphi(rB_X)$ is relatively $\sigma(X, \mathcal{P}(X))$-compact for every $0 < r < 1$. Then $C_\varphi : H_b(B_X) \to H_b(B_X)$ is mean ergodic if and only if it is power bounded (eq. φ has B_X-stable orbits).

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The Tsirelson space T^* satisfies the assumption.

$H_b(B)$: ME $\not\Rightarrow$ UME

Lemma (Köthe II)

Let $(T_n)_n$ be a sequence of equicontinuous operators on a lcHs. If it converges pointwise to an operator T on some dense set, then $(T_n)_n$ is pointwise convergent to T in the whole space.

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Theorem (A. Defant, D. García, M. Maestre, P. Sevilla-Peris)

The set of monomials generates a dense subspace of $H_b(B_{c_0})$.

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The set of monomials generates a dense subspace of $H_b(B_{c_0})$.

Example

The operator $C_F : H_b(B_{c_0}) \to H_b(B_{c_0})$ is mean ergodic but not uniformly mean ergodic.

$H_b(B)$: Suficient conditions for UME

Proposition

Let $\varphi : B \to B$ be holomorphic so that for every $0 < t < 1$ there is $0 < \rho < t$ such that

$$\varphi(tB) \subseteq \rho B.$$

Then $C_\varphi^n \to C_0$ uniformly on the bounded sets of $H_b(B)$ and C_φ is uniformly mean ergodic.

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The following polynomial in $\mathcal{P}(^2\ell_2, \ell_2)$ satisfies the assumption

$$P(x_1, x_2, x_3, \dots) = (x_1^2, x_2^2, x_3^2, \dots).$$

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Remark

In particular, if $\varphi(0) = 0$ and there is $0 < r < 1$ such that $\varphi(B) \subseteq rB$ we obtain that C_φ is uniformly mean ergodic.

$H_b(B)$: The Hilbert space case

Proposition

Let $\varphi : B_H \to B_H$ be holomorphic such that

$$\varphi(B_H) \subseteq rB_H \text{ for some } 0 < r < 1.$$

Then for the unique $a \in B_H$ such that $\varphi(a) = a$ we have $C_\varphi^n \to C_a$ uniformly on the bounded sets of $H_b(B_H)$ and C_φ is uniformly mean ergodic.

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Then for the unique $a \in B_H$ such that $\varphi(a) = a$ we have $C_\varphi^n \to C_a$ uniformly on the bounded sets of $H_b(B_H)$ and C_φ is uniformly mean ergodic.

The existence and uniqueness of the point a is given by the Earle-Hamilton fixed point theorem.

$H^{\infty}(B)$: Power bounded and Toplogizable

Let $\varphi : B \to B$ be a holomorphic map. We have

$$\|C_{\varphi}^{n}(f)\| = \sup_{x \in B} |f(\varphi^{n}(x))| \leq \sup_{x \in B} |f(x)| = \|f\|$$

for every $n \in \mathbb{N}$ and $f \in H^{\infty}(B)$.

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for every $n \in \mathbb{N}$ and $f \in H^{\infty}(B)$.

Proposition

Every composition operator defined in $H^{\infty}(B)$ is power bounded and topologizable.

$H^\infty(B)$: Suficient conditions for UME

Proposition

Let $\varphi : B \to B$ be holomorphic such that $\varphi(B) \subseteq rB$ for some $0 < r < 1$ and $\varphi(0) = 0$. Then $C_\varphi^n \to C_0$ uniformly on the bounded sets of $H^\infty(B)$ and C_φ is uniformly mean ergodic.

$H^\infty(B)$: The Hilbert space case

Proposition

Let $\varphi : B_H \to B_H$ be holomorphic such that

$$\varphi(B_H) \subseteq rB_H \text{ for some } 0 < r < 1.$$

Then for the unique $a \in B_H$ such that $\varphi(a) = a$ we have $C_\varphi^n \to C_a$ uniformly on the bounded sets of $H^\infty(B_H)$ and C_φ is uniformly mean ergodic.

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