



# WFCA22

Workshop on Functional and Complex Analysis

from June 20th to 23rd in Valladolid (Spain)



---

**Universidad de Valladolid**



# Contents

<b>About</b>	<b>5</b>
WFCA22 . . . . .	5
Topics . . . . .	5
Scientific Committee . . . . .	5
Organizing Committee . . . . .	5
<b>Schedule</b>	<b>6</b>
Monday, June 20th . . . . .	6
Tuesday, June 21st . . . . .	7
Wednesday, June 22nd . . . . .	8
Thursday, June 23rd . . . . .	9
<b>List of Abstracts</b>	<b>10</b>
V. Asensio - Global wave front sets in ultradifferentiable classes . . . . .	10
J. Bonet - Toeplitz operators on weighted spaces of analytic functions . . . . .	10
S. A. Carrillo - $q$ -Nagumo norms and formal solutions of singularly perturbed $q$ -difference equations . . . . .	10
C. Fernández - Dynamics and spectra of composition operators on $\mathcal{S}(\mathbb{R})$ . . . . .	11
A. Galbis - Wigner transform and quasicrystals . . . . .	11
J. Galindo - Ergodic multiplication operators on the Fourier algebra . . . . .	11
P. Galindo - Some results on the Bloch space over the unit ball of a Hilbert space	12
J. Jiménez-Garrido - A comparison of two ways to generalize Denjoy-Carleman ultradifferentiable classes . . . . .	12
E. Jordá - On supercyclic composition operator . . . . .	12
D. Jornet - Compact weighted composition operators on spaces of holomorphic functions on Banach spaces . . . . .	13
T. Kalmes - Quantitative Runge type approximation theorems for kernels of partial differential operators . . . . .	13
T. Lambay & S. Nicolay - Generalized interpolation: a functorial point of view	14
F. Martínez-Giménez - Dynamics of skew-products of differential operators . . . . .	15
C. Mele - On the spaces $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$ and $\mathcal{O}_{C,\omega}(\mathbb{R}^N)$ . . . . .	15
I. Miguel-Cantero - Optimal flat functions in Carleman-Roumieu ultraholomor- phic classes in sectors . . . . .	16
A. Molla - Pointwise regularity and multifractal analysis on Lie groups . . . . .	16
J. Mozo-Fernández - Nilpotent singularities of holomorphic foliations. New results on two problems . . . . .	17
L. Neyt - Sequence space representations for translation-modulation invariant function and distribution spaces . . . . .	17

D. N. Nenning - The Borel map in the mixed Beurling setting . . . . .	18
A. Oliario - Global regularity in ultradifferentiable spaces for non hypoelliptic PDE . . . . .	18
A. Peris - Wild dynamics and periods for operators on Hilbert spaces . . . . .	19
A. Rainer - Nonlinear conditions for ultradifferentiability . . . . .	19
A. Rodríguez-Arenas - Uniformly ergodic measures on locally compact groups	20
Ó. Roldán - Norm-attainment questions for nuclear operators and tensors . .	20
D. Santacreu - Mean ergodic composition operators on spaces of holomorphic functions . . . . .	20
F. Sanz-Sánchez - Real Turrittin's Theorem and applications to trajectories of vector fields . . . . .	21
G. Schindl - On the maximal extension in the mixed ultradifferentiable weight sequence setting . . . . .	21
P. Speissegger - The real Gamma and Zeta functions are jointly o-minimal . .	22
J. Vindas - Factorization theorems in Denjoy-Carleman classes associated to representations of $(\mathbb{R}^d, +)$ . . . . .	22
J. Wengenroth - Continuously differentiable functions on compact sets . . . . .	23
<b>List of Participants</b>	<b>24</b>
<b>Useful Information</b>	<b>26</b>
<b>Partner Institutions and Sponsors</b>	<b>28</b>

## WFCA22

The Workshop FCA22, to be held in Valladolid (Spain) from June 20th to 23rd, will bring together experts in the field of Functional and Complex Analysis. The workshop will have a number of talks, stimulating further discussions. Well known experts on the area will meet younger researchers with the aim of improving the exchange of ideas and creating new scientific collaborations.

The Organizing and Scientific Committees have the commitment to make this meeting profitable and fruitful, please do not hesitate to contact them for any question or advice.

You can address any enquiry, question or suggestion to [wfca22@gmail.com](mailto:wfca22@gmail.com)

## Topics

- Operator Theory.
- Spaces and classes of functions and distributions, and operators defined between them.
- Asymptotic analysis and summability for functional equations in the complex domain.
- Quasianalytic classes and O-minimality.

## Scientific Committee

José Bonet	(Polytechnic University of Valencia, Spain)
Fernando Sanz	(University of Valladolid, Spain)
Javier Sanz	(University of Valladolid, Spain)
Jasson Vindas	(Ghent University, Belgium)

## Organizing Committee

Javier Jiménez-Garrido	(University of Cantabria, Spain)	<a href="mailto:jimenezjj@unican.es">jimenezjj@unican.es</a>
Alberto Lastra	(University of Alcalá, Spain)	<a href="mailto:alberto.lastra@uah.es">alberto.lastra@uah.es</a>
María Martín Vega	(University of Valladolid, Spain)	<a href="mailto:maria.martin.vega@uva.es">maria.martin.vega@uva.es</a>
Ignacio Miguel	(University of Valladolid, Spain)	<a href="mailto:ignacio.miguel@uva.es">ignacio.miguel@uva.es</a>
Javier Sanz	(University of Valladolid, Spain)	<a href="mailto:javier.sanz.gil@uva.es">javier.sanz.gil@uva.es</a>

# Schedule

## Monday, June 20th

9:30-10:10	<b>Registration and welcome remarks</b>	
10:15-10:55	<b>Pablo Galindo</b> Universidad de Valencia, Spain	Some results on the Bloch space over the unit ball of a Hilbert space
11:00-11:30	<b>Coffee</b>	
11:30-12:10	<b>Alessandro Oliaro</b> University of Torino, Italy	Global regularity in ultradifferentiable spaces for non hypoelliptic PDE
12:15-12:55	<b>Thomas Kalmes</b> Chemnitz Technical University, Germany	Quantitative Runge type approximation theorems for kernels of partial differential operators
13:00-15:00	<b>Lunch</b>	
15:00-15:40	<b>Alfred Peris</b> IUMPA & Universitat Politècnica de València, Spain	Wild dynamics and periods for operators on Hilbert spaces
15:45-16:25	<b>Félix Martínez-Giménez</b> IUMPA & Universitat Politècnica de València, Spain	Dynamics of skew-products of differential operators
16:30-17:00	<b>Coffee</b>	
17:00-17:40	<b>Thomas Lamby</b> University of Liège, Belgium	Generalized interpolation: a functorial point of view (part 1)
17:45-18:25	<b>Samuel Nicolay</b> University of Liège, Belgium	Generalized interpolation: a functorial point of view (part 2)

## Tuesday, June 21st

9:30–10:10	<b>José Bonet</b> IUMPA & Universitat Politécnica de València, Spain	Toeplitz operators on weighted spaces of analytic functions
10:15–10:55	<b>Enrique Jordá</b> IUMPA & Universitat Politécnica de València, Spain	On supercyclic composition operator
11:00–11:30	<b>Coffee</b>	
11:30–12:10	<b>Carmen Fernández</b> Universitat de València, Spain	Dynamics and spectra of composition operators on $\mathcal{S}(\mathbb{R})$
12:15–12:55	<b>Antonio Galbis</b> Universitat de València, Spain	Wigner transform and quasicrystals
13:00–15:00	<b>Lunch</b>	
15:00–15:40	<b>Jochen Wengenroth</b> Universität Trier, Germany	Continuously differentiable functions on compact sets
15:45–16:25	<b>Arman Molla</b> University of Liège, Belgium	Pointwise regularity and multifractal analysis on Lie groups
16:30–17:00	<b>Coffee</b>	
17:00–17:40	<b>Daniel Santacreu</b> IUMPA & Universitat Politécnica de València, Spain	Mean ergodic composition operators on spaces of holomorphic functions
17:45–18:25	<b>Alberto Rodríguez-Arenas</b> Universitat Jaume I, Spain	Uniformly ergodic measures on locally compact groups
20:30	<b>Conference Dinner</b>	

## Wednesday, June 22nd

9:30–10:10	<b>Jasson Vindas</b> Ghent University, Belgium	Factorization theorems in Denjoy-Carleman classes associated to representations of $(\mathbb{R}^d, +)$
10:15–10:55	<b>David Jornet</b> IUMPA & Universitat Politècnica de València, Spain	Compact weighted composition operators on spaces of holomorphic functions on Banach spaces
11:00–11:30	<b>Coffee</b>	
11:30–12:10	<b>Lenny Neyt</b> Ghent University, Belgium	Sequence space representations for translation-modulation invariant function and distribution spaces
12:15–12:55	<b>Vicente Asensio</b> IUMPA & Universitat Politècnica de València, Spain	Global wave front sets in ultradifferentiable classes
13:00–15:00	<b>Lunch</b>	
15:00–15:40	<b>Armin Rainer</b> University of Vienna, Austria	Nonlinear conditions for ultradifferentiability
15:45–16:25	<b>Gerhard Schindl</b> University of Vienna, Austria	On the maximal extension in the mixed ultradifferentiable weight sequence setting
16:30–17:00	<b>Coffee</b>	
17:00–17:40	<b>David Nicolas Nenning</b> University of Vienna, Austria	The Borel map in the mixed Beurling setting
17:45–18:25	<b>Javier Jiménez-Garrido</b> Universidad de Cantabria & IMUVA, Spain	A comparison of two ways to generalize Denjoy-Carleman ultradifferentiable classes
20:00	<b>Guided city tour</b>	



## Thursday, June 23rd

9:30–10:10	<b>Sergio Alejandro Carrillo</b> Universidad Sergio Arboleda, Colombia	$q$ -Nagumo norms and formal solutions of singularly perturbed $q$ -difference equations
10:15–10:55	<b>Óscar Roldán</b> Universitat de València	Norm-attainment questions for nuclear operators and tensors
11:00–11:30	<b>Coffee</b>	
11:30–12:10	<b>Jorge Galindo</b> Universitat Jaume I, Spain	Ergodic multiplication operators on the Fourier algebra
12:15–12:55	<b>Claudio Mele</b> Università del Salento, Italy	On the spaces $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$ and $\mathcal{O}_{C,\omega}(\mathbb{R}^N)$
13:00–15:00	<b>Lunch</b>	
15:00–15:40	<b>Fernando Sanz-Sánchez</b> Universidad de Valladolid, Spain	Real Turrittin's Theorem and applications to trajectories of vector fields
15:45–16:25	<b>Patrick Speissegger</b> McMaster University, Canada	The real Gamma and Zeta functions are jointly o-minimal
16:30–17:00	<b>Coffee</b>	
17:00–17:40	<b>Jorge Mozo-Fernández</b> Universidad de Valladolid, Spain	Nilpotent singularities of holomorphic foliations. New results on two problems
17:45–18:25	<b>Ignacio Miguel-Cantero</b> Universidad de Valladolid & IMUVA, Spain	Optimal flat functions in Carleman-Roumieu ultraholomorphic classes in sectors
18:30	<b>Closing</b>	

# List of Abstracts

## Global wave front sets in ultradifferentiable classes

*Vicente Asensio*, Instituto Universitario de Matemática Pura y Aplicada IUMPA & Universitat Politècnica de València, Spain

The aim of this talk is to introduce a global wave front set using Weyl quantizations of global pseudodifferential operators of infinite order in ultradifferentiable classes of Beurling type, extending previous results by Rodino and Wahlberg. We will see that, in many cases, this wave front set coincides with the Gabor wave front set, given in terms of Gabor frames or the short-time Fourier transform, studied by Boiti, Jornet, and Oliaro in the ultradifferential setting. Applications of this wave front set to the regularity of pseudodifferential operators are provided.

Joint work with C. Boiti, D. Jornet, and A. Oliaro.

## Toeplitz operators on weighted spaces of analytic functions

*José Bonet*, Instituto Universitario de Matemática Pura y Aplicada IUMPA & Universitat Politècnica de València, Spain

We present several results about continuity and compactness of Toeplitz operators defined on weighted Banach spaces  $H_v^\infty$  of analytic functions with sup-norms. In particular, we study the case of exponential weights  $v(r) = \exp(-\alpha/(1-r)^\beta)$ ,  $\alpha, \beta > 0$  on the unit disc of the complex plane.

Joint work with Wolfgang Lusky and Jari Taskinen.

## $q$ -Nagumo norms and formal solutions of singularly perturbed $q$ -difference equations

*Sergio Alejandro Carrillo*, Universidad Sergio Arboleda, Colombia

The aim of this talk is to give a  $q$ -analogous to the classical Nagumo norms, familiar in the framework of analytic PDEs. These norms turn out to be useful in the study of the divergence rate of formal power series solutions of some families of analytic  $q$ -difference equations in several variables. In fact, we will find such rate for equations

involving irregular singularities (normal crossings) and perturbation parameters. Finally, by studying confluence, i.e., letting  $q \rightarrow 1$ , we recover the Gevrey order of formal solutions of PDEs with the same singular type.

Joint work with Alberto Lastra.

## Dynamics and spectra of composition operators on $\mathcal{S}(\mathbb{R})$

*Carmen Fernández*, [Universitat de València, Spain](#)

Some results concerning the behaviour of the iterates of composition operators on the Schwartz space and their spectra will be presented, mainly focusing on polynomial symbols.

Joint work with A. Galbis and E. Jordá.

## Wigner transform and quasicrystals

*Antonio Galbis*, [Universitat de València, Spain](#)

A Fourier quasicrystal is a discrete measure whose Fourier transform is also a discrete measure. We prove that a tempered distribution whose Wigner transform is supported on a product of two uniformly discrete sets is a quasicrystal. This suggests that techniques from time-frequency analysis could possibly be useful tools in the study of such structures.

Joint work with P. Boggiatto, C. Fernández, A. Oliaro.

## Ergodic multiplication operators on the Fourier algebra

*Jorge Galindo*, [Universitat Jaume I, Spain](#)

This talk is a follow-up of Alberto Rodríguez's talk *Uniformly ergodic measures on locally compact groups*. In his talk, he discussed the ergodic properties of convolution operators  $\lambda_1^0(\mu)f = \mu * f$  where  $\mu$  is a probability measure on a locally compact Abelian group  $G$  and  $\lambda_1^0(\mu)$  is seen as an operator on the augmentation ideal  $L_1^0(G)$  of the group algebra  $L_1(G)$ . In this talk we will see how most of these results can be deduced from the ergodic properties of multiplication operators  $M_0(\phi)(u) = \phi \cdot u$ , where  $\phi$  is an element of the Fourier-Stieltjes algebra  $B(G)$  of an amenable locally compact group  $G$  and  $M_0(\phi)$  is seen as an operator on the augmentation ideal  $A_0(G)$  of the Fourier algebra  $A(G)$ ,  $A_0(G) = \{u \in A(G) : u(1) = 0\}$ . Our discussion will include the introduction of the

algebras  $A(G)$  and  $B(G)$  and a description of how the arguments required for  $M_0(\phi)$  parallel, and how they differ from, those needed for  $\lambda_1^0(\mu)$  beyond, of course, the Abelian case where the operators  $M_0(\phi)$  and  $\lambda_1^0(\mu)$  are unitarily equivalent via Fourier-Stieltjes transforms.

This talk reports on ongoing joint work with Enrique Jordá and Alberto Rodríguez.

## **Some results on the Bloch space over the unit ball of a Hilbert space**

*Pablo Galindo*, Universidad de Valencia, Spain

We will comment on some results concerning the Bloch space of analytic functions of many variables, particularly of a Hilbert space variable. Among them, its maximality within the class of Banach spaces of analytic functions on the unit ball of a Hilbert space  $H$  that are invariant under automorphisms of the ball  $B_H$ .

Joint work with Wolfgang Lusky and Jari Taskinen.

## **A comparison of two ways to generalize Denjoy-Carleman ultradifferentiable classes**

*Javier Jiménez-Garrido*, Universidad de Cantabria & Instituto de Matemáticas de la Universidad de Valladolid IMUVA, Spain

In the past decade and simultaneously, two ways to generalize Denjoy-Carleman ultradifferentiable classes have appeared. We will show that the ultradifferentiable classes of functions defined and studied in several papers by S. Pilipović, N. Teofanov and F. Tomić are special cases of the classes defined by weight matrices in the sense of A. Rainer and G. Schindl. In addition, we will detail how the results obtained in the general framework of the latter theory can be applied to the former.

Joint work with David Nicolas Nenning and Gerhard Schindl.

## **On supercyclic composition operator**

*Enrique Jordá*, Instituto Universitario de Matemática Pura y Aplicada IUMPA & Universitat Politècnica de València, Spain

Let  $E \hookrightarrow (C(X), \tau_p)$  be a Fréchet space, where  $X$  is a locally compact topological space, and let  $\varphi : X \rightarrow X$  a symbol, i.e. a function such that  $e \circ \varphi \in E$  for all  $e \in E$ . We study

when  $C_\varphi$  is supercyclic in  $E$ ,  $(E, \omega)$  or  $(E, \tau_p)$ . We obtain the following results:

- (1) No composition operator  $C_\varphi$  is supercyclic in  $(A(\mathbb{D}), \tau_p)$ .
- (2) No composition operator  $C_\varphi$  is supercyclic in  $(H(\mathbb{D}) \setminus \{0\}, \omega)$  or in  $H(\mathbb{C} \setminus \{0\}, \omega)$ .
- (3) For any  $m \in \mathbb{N}$ , a composition operator  $C_\varphi$  in  $C^m(\mathbb{R})$  is weakly supercyclic if and only if it is mixing.
- (4) A composition operator  $C_\varphi$  in  $C(\mathbb{R})$  is supercyclic if and only if it is mixing.

The results (1) and (2) belong to a joint work with M. J. Beltrán-Meneu and M. Murillo-Arcila, and (3) and (4) to a joint work with A. A. Albanese and C. Mele.

## Compact weighted composition operators on spaces of holomorphic functions on Banach spaces

*David Jornet*, [Universitat Politècnica de València & Instituto Universitario de Matemática Pura y Aplicada IUMPA, Spain](#)

Given an infinite dimensional Banach space  $X$  and its open unit ball  $B$ , we study when the weighted composition operator  $C_{\psi, \varphi}$  is compact in the space of all bounded analytic functions  $H^\infty(B)$ , and when it is bounded, reflexive, Montel and (weakly) compact in the space of analytic functions of bounded type  $H_b(B)$ . The study is given in terms of properties of the weight  $\psi$  and the symbol  $\varphi$ .

Joint work with José Bonet, Daniel Santacreu and Pablo Sevilla-Peris.

## Quantitative Runge type approximation theorems for kernels of partial differential operators

*Thomas Kalmes*, [Chemnitz Technical University, Germany](#)

A well known consequence of Runge's classical theorem on rational approximation is that for open subsets  $X_1 \subseteq X_2$  of the complex plane  $\mathbb{C}$ , every holomorphic function on  $X_1$  can be approximated by a holomorphic function on  $X_2$  with respect to the compact-open topology, precisely when  $X_2$  does not contain a compact component of  $\mathbb{C} \setminus X_1$ . In the 1950s, this approximation theorem has been generalized independently by Lax [4] and Malgrange [5] from holomorphic functions, i.e. functions in the kernel of the Cauchy-Riemann operator, to kernels of elliptic constant coefficient differential operators.

Recently, there has been an renewed interest in such approximation results, namely in so-called quantitative Runge type approximation results for elliptic operators of second

order on the one hand (cf. [6]) and generalizations of the Lax-Malgrange Theorem to certain classes of non-elliptic operators on the other hand (cf. [1], [2], [3]).

In our talk, we show how recent results on Vogt's and Wagner's topological invariant  $(\Omega)$ , introduced in [7], for kernels of partial differential operators can be used to obtain quantitative Runge type approximation results for elliptic operators of arbitrary order, non-degenerate parabolic operators and the one dimensional wave operator.

Joint work with A. Debrouwere (University of Brussels).

## References

- [1] A. Enciso, M.A. García-Ferrero, D. Peralta-Salas, *Approximation theorems for parabolic equations and movement of local hot spots*. Duke Math. J. 168(5):897–939, 2019.
- [2] A. Enciso, D. Peralta-Salas, *Approximation theorems for the Schrödinger equation and quantum vortex reconnection*. Comm. Math. Phys. 387(2):1111–1149, 2021.
- [3] T. Kalmes, *An approximation theorem of Runge type for certain non-elliptic partial differential operators*. Bull. Sci. Math. 170, Paper No. 103012, 26 pp. 2021.
- [4] P. D. Lax, *A stability theorem for solutions of abstract differential equations, and its application to the study of the local behavior of solutions of elliptic equations*. Comm. Pure Appl. Math. 9:747–766, 1956.
- [5] B. Malgrange, *Existence et approximation des solution des équations aux dérivées partielles et des équation de convolution*. Ann. Inst. Fourier (Grenoble) 6:71–355, 1955-56.
- [6] A. Rüländ, M. Salo, *Quantitative Runge approximation and inverse problems*. Int. Math. Res. Not. IMRN 20:6216–6234, 2019.
- [7] D. Vogt, M.J. Wagner, *Charakterisierung der Quotientenräume von  $s$  und eine Vermutung von Martineau*. Studia Math. 67(3):225–240, 1980.

## Generalized interpolation: a functorial point of view

*Thomas Lamby and Samuel Nicolay, University of Liège, Belgium*

Many spaces, such as the Besov spaces  $B_{p,q}^s$ , naturally arise through the (real) interpolation theory:  $B_{p,q}^s = [H_p^t, H_p^u]_{\alpha,q}$  roughly means that this Besov space “lies” between the Sobolev spaces  $H_p^t$  and  $H_p^u$ , with  $s = (1 - \alpha)t + \alpha u$ . A classical generalization of these interpolation methods has been introduced in the eighties. The idea is to replace the linear formula  $(1 - \alpha)t + \alpha u$  by a more general function  $\alpha \mapsto f(\alpha)$ . As the interpolation theory can be expressed using the language of categories, it is natural to do the same with the generalized interpolation spaces. This is the purpose of these talks.

We first discuss the existing relations between the Boyd functions and the admissible sequences, with a particular interest to the Boyd indices. These notions are intended to be tools in order to generalize some functional spaces. We then define interpolation functors depending on Boyd functions from the category of compatible normed vector spaces to the category of normed vector spaces. Next, we generalize some classic results

of interpolation theory and apply them to some classical functional spaces.

This talk is divided into two parts. The first talk will be more about the general notions, while the second one will explore how the classical result can be adapted in this generalized setting.

## Dynamics of skew-products of differential operators

*Félix Martínez-Giménez*, Instituto Universitario de Matemática Pura y Aplicada IUMPA & Universitat Politècnica de València, Spain

Let  $X$  be a separable complex Fréchet space,  $(A, \mu)$  a probability space,  $T : X \rightarrow X$  a linear and continuous operator,  $f : A \rightarrow A$  an ergodic map with respect to the measure  $\mu$  and  $h : A \rightarrow \mathbb{C}$  a  $L^1(\mu)$  function. The map

$$P : A \times X \rightarrow A \times X \text{ defined by } P(a, x) = (f(a), h(a)Tx)$$

is said to be a **skew-product** of the operator  $T$ .

Skew-products provide a rich source of dynamical systems whose dynamics vary as the state of the system evolves. One may think of a skew-product as a dynamical system dependent on a parameter that is perturbed as the system evolves in a particular way.

In this talk we present some results concerning the dynamics of skew-products associated to operators  $T : H(\mathbb{C}) \rightarrow H(\mathbb{C})$  such that they commute with the differentiation operator. We show the skew-products of  $T$  are topologically transitive and even chaotic, in the sense of Devaney, under several (weak) assumptions on  $f$  and  $h$ .

Joint work with H. Méndez, F. Rodenas and A. Peris.

## On the spaces $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$ and $\mathcal{O}_{C,\omega}(\mathbb{R}^N)$

*Claudio Mele*, Università del Salento, Italy

In the last years the attention has focused on the space  $\mathcal{S}_\omega(\mathbb{R}^N)$  of the ultradifferentiable rapidly decreasing functions of Beurling type, as defined by Björck. Motivated by this line of research, in [1, 2] the authors introduced and studied the space  $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$  of the slowly increasing functions and the space  $\mathcal{O}_{C,\omega}(\mathbb{R}^N)$  of the very slowly increasing functions in the setting of ultradifferentiable function spaces of Beurling type. For instance in [1, 2] the authors have proved that  $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$  is the space of the multipliers of  $\mathcal{S}_\omega(\mathbb{R}^N)$  and of its dual  $\mathcal{S}'_\omega(\mathbb{R}^N)$  and that the strong dual  $\mathcal{O}'_{C,\omega}(\mathbb{R}^N)$  of  $\mathcal{O}_{C,\omega}(\mathbb{R}^N)$  is the space of the convolutors of  $\mathcal{S}_\omega(\mathbb{R}^N)$  and of its dual  $\mathcal{S}'_\omega(\mathbb{R}^N)$ .

In the present talk, we show that  $\mathcal{O}_{C,\omega}(\mathbb{R}^N)$  is a sequentially retractive Montel (LF)-space. Moreover, we prove that  $(\mathcal{O}_{M,\omega}(\mathbb{R}^N), \cdot)$ ,  $(\mathcal{S}_\omega(\mathbb{R}^N), \cdot)$  are multiplication topological algebras, while  $(\mathcal{S}_\omega(\mathbb{R}^N), \star)$  and  $(\mathcal{O}'_{C,\omega}(\mathbb{R}^N), \star)$  are convolution topological algebras.

Joint work with Angela A. Albanese

### References

- [1] A.A. Albanese, C. Mele, *Multipliers on  $\mathcal{S}_\omega(\mathbb{R}^N)$* , J. Pseudo-Differ. Oper. Appl. **12** (2021), Article 35.
- [2] A.A. Albanese, C. Mele, *Convolutors on  $\mathcal{S}_\omega(\mathbb{R}^N)$* , RACSAM **115** (2021), Article 157.
- [3] A.A. Albanese, C. Mele, *Multiplication and convolution topological algebras in spaces of  $\omega$ -ultradifferentiable function of Beurling type*, in: Recent Advances in Mathematical Analysis, Trends in Mathematics (to appear).
- [4] A.A. Albanese, C. Mele, *On the space  $\mathcal{O}_{C,\omega}(\mathbb{R}^N)$  and its dual*, preprint 2022.

## Optimal flat functions in Carleman-Roumieu ultraholomorphic classes in sectors

*Ignacio Miguel Cantero*, Universidad de Valladolid & Instituto de Matemáticas de la Universidad de Valladolid IMUVA, Spain

A general procedure is presented in order to obtain linear continuous extension operators, right inverses of the Borel map, whenever optimal flat functions are available in Carleman-Roumieu ultraholomorphic classes, defined on sectors and in terms of regular weight sequences in the sense of Dyn'kin. For a family of regular sequences, including the well-known q-Gevrey case, we construct such optimal flat functions in arbitrary sectors of the Riemann surface of the logarithm.

Only very recently we have been able to construct optimal flat functions in Carleman-Roumieu ultraholomorphic classes for general weight sequences  $\mathbb{M}$  (not even regular) and in sectors whose opening is limited in terms of the index  $\gamma(\mathbb{M})$  of V. Thilliez. This step forward is due to the equivalence between the classical conditions (M3) of H. Komatsu and a condition appearing in a work of M. Langenbruch which has been recovered in a preprint of D. N. Nenning, A. Rainer and G. Schindl.

Joint work with J. Jiménez-Garrido, J. Sanz and G. Schindl.

## Pointwise regularity and multifractal analysis on Lie groups

*Arman Molla*, University of Liège, Belgium

Multifractal analysis is a very recent branch of mathematics studying the pointwise behavior of a locally bounded function to determine its Holder exponent and its monofractal



or multifractal nature. It has already been widely studied for functions defined in the Euclidean space, with still a lot of unanswered questions. However, very few was done in a more general context. For example, it could be useful to define such concepts on some manifolds.

In this talk, we will extend the concept of pointwise, local and uniform Holder spaces in a unified way to the case of unimodular Lie groups and give a few characterizations of these spaces. We will also briefly compare these notions with what was already done in this framework. A particular attention will be given to compact Lie groups where a generalization of the Weierstrass continuous nowhere differentiable function can be defined.

## **Nilpotent singularities of holomorphic foliations. New results on two problems**

*Jorge Mozo Fernández, Universidad de Valladolid*

Nilpotent singularities of germs of holomorphic foliations in  $\mathbb{C}^2$  have been studied by different authors since the 1980s. One of the main problems has been the analytic classification of these foliations through their projective holonomy group. Almost all cases are studied, except one: when a Poincaré-Dulac type singularity appears. In a joint work with Percy Pernández (PUCP, Lima, Perú), this case is also studied, and a new rigidity formal-analytic result is obtained.

A second problem concerns foliations on the projective plane, with only one singularity of maximal Milnor number. Following some results from Claudia R. Alcántara (Guanajuato University, México), a family of foliations satisfying previous property has, as only singular point, a nilpotent one, of the generalized saddle-node type. In a work in progress with Claudia R. Alcántara, using index theorems, we can prove that the formal separatrix of these foliations is, in fact, convergent. This is an application of global techniques to solve local problems.

In this talk I will try to give an overview of these two results.

## **Sequence space representations for translation-modulation invariant function and distribution spaces**

*Lenny Neyt, Ghent University, Ghent, Belgium*

The representation of function and distribution spaces by sequence spaces is a central topic in functional analysis that goes back to the pioneering work of Valdivia and Vogt and is closely connected to the isomorphic classification of such spaces. In particular, there has been a considerable interest in the sequence space representations of the spaces

appearing in Schwartz's classical distribution theory. In this talk, we demonstrate how one may unify and generalize many of these known representations by providing sequence representations for test function and distribution spaces defined via a broad class of translation-modulation invariant Banach spaces. Our proof is based on the Gabor frame characterization of the aforementioned spaces and the use of the Pełczyński decomposition method. As an application of our results, we present the isomorphic classification of the spaces  $\mathcal{D}_{L^{p_1} \widehat{\otimes}_\pi L^{p_2}}$ ,  $1 < p_1, p_2 < \infty$ , thereby showing that these spaces are genuinely new and distinct.

## The Borel map in the mixed Beurling setting

*David Nicolas Nenning, University of Vienna, Austria*

The Borel map  $j_0^\infty : C^\infty(\mathbb{R}) \rightarrow \mathbb{C}^\mathbb{N}$  assigns to a smooth function its sequence of derivatives (at 0). It is a classical result, that the Borel map is surjective in this setting. Analogous questions for ultradifferentiable classes defined via weight sequences and weight functions have been studied extensively in the last decades. More precisely it was characterized, in terms of the defining weights, when the image of the Borel map covers a sequence space defined via the same weight structure. We give new characterizations for Beurling-type classes defined via weight matrices and recover classical results for weight sequences and weight functions as special cases.

This is joint work with Armin Rainer and Gerhard Schindl.

## Global regularity in ultradifferentiable spaces for non hypoelliptic PDE

*Alessandro Oliaro, University of Torino, Italy*

The problem of global regularity for partial differential operators was first introduced by Shubin in the frame of Schwartz functions and tempered distributions; a linear operator  $A : \mathcal{S}' \rightarrow \mathcal{S}'$  is said to be regular if the conditions  $u \in \mathcal{S}'$ ,  $Au \in \mathcal{S}$  imply that  $u \in \mathcal{S}$ . Shubin formulates an hypoellipticity condition (in his global pseudodifferential calculus), that is sufficient to have regularity of the corresponding operator. On the other hand, such hypoellipticity is far to be necessary, as there are several examples of operators which are not hypoelliptic but are globally regular (such as the Twisted Laplacian). The problem of characterizing global regularity for classes of operators is quite hard. Even in very particular cases (as for ordinary differential operators with polynomial coefficients) necessary and sufficient conditions for global regularity are not known.

In this talk we present some results on global regularity of (non hypoelliptic) linear partial differential operators with polynomial coefficients in non isotropic ultradifferentiable

classes of Beurling type, using techniques from time-frequency analysis; the techniques are related to the transformations of the operator itself by a quadratic representation of Wigner type.

Joint works with E. Buzano, C. Boiti, D. Jornet, C. Mele.

### References

- [1] C. Boiti, D. Jornet, A. Oliaro, *Regularity of partial differential operators in ultradifferentiable spaces and Wigner type transforms*, J. Math. Anal. Appl. 446 (2017), 920-944.
- [2] A. Enciso, D. Peralta-Salas, E. Buzano, A. Oliaro, *Global regularity of second order twisted differential operators*, J. Differential Equations 268, 7364–7416 (2020).
- [3] C. Mele, A. Oliaro, *Regularity of global solutions of partial differential equations in non isotropic ultradifferentiable spaces via time-frequency methods*, J. Differential Equations 286, 821–855 (2021).

## Wild dynamics and periods for operators on Hilbert spaces

*Alfred Peris*, Institut Universitari de Matemàtica Pura i Aplicada IUMPA & Universitat Politècnica de València, Spain

Since Sharkovsky's characterization in 1964 of possible periods for a continuous map on an interval, the study of periods that certain dynamical systems allow, depending on the structure of the phase space or the map, has attracted a lot of attention. We will present a complete characterization of all possible periods that a Devaney chaotic operator on the Hilbert space can present.

Joint work with J.A. Conejero, F. Martínez-Giménez and F. Rodenas.

### References

- [1] J.A. Conejero, F. Martínez-Giménez, A. Peris, and F. Rodenas, *Sets of periods for chaotic linear operators*, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales - Serie A: Matemáticas 115(2) (2021).

## Nonlinear conditions for ultradifferentiability

*Armin Rainer*, University of Vienna, Austria

A remarkable theorem of Joris states that a function  $f : \mathbb{R}^d \rightarrow \mathbb{C}$  is of class  $C^\infty$  if two coprime powers of  $f$  are of class  $C^\infty$ . Thilliez showed that this result carries over to suitable Denjoy–Carleman classes of Roumieu type, where  $d = 1$ . Refining Thilliez' methods, I will show that the theorem of Joris is valid for all ultradifferentiable classes  $\mathcal{C}$  with basic stability properties (including Braun–Meise–Taylor classes and classes defined by weight matrices of Beurling and Roumieu type), in all dimensions (even on infinite dimensional Banach spaces and convenient vector spaces), no matter if quasianalytic or

not. The core of the proof is based on uniform unidirectional holomorphic approximation and almost analytic extension.

With this result at disposal, we then derive a full characterization of the analytic germs  $\Phi : (\mathbb{K}, 0) \rightarrow (\mathbb{K}^n, 0)$  (where  $\mathbb{K}$  is  $\mathbb{R}$  or  $\mathbb{C}$ ) such that  $\Phi \circ f \in \mathcal{C}$  implies  $f \in \mathcal{C}$ , for all continuous function germs  $f : (\mathbb{R}^d, 0) \rightarrow (\mathbb{K}, 0)$ , in terms of a condition on the support of the Taylor series of  $\Phi$ .

Joint work with David Nicolas Nenning and Gerhard Schindl.

## Uniformly ergodic measures on locally compact groups

*Alberto Rodríguez-Arenas*, [Universitat Jaume I, Spain](#)

We completely characterize the uniform mean ergodicity and the asymptotic behaviour of the convolution operator by a probability measure in  $L^1(G)$ , with  $G$  a locally compact Abelian group. These characterizations depend on properties of the measure and the group. We also study the particular cases when  $G$  is and isn't compact.

## Norm-attainment questions for nuclear operators and tensors

*Óscar Roldán*, [Universitat de València, Spain](#)

Let  $X$  and  $Y$  be two Banach spaces. The set of norm-attaining operators from  $X$  to  $Y$ , and whether or not it can be dense, has been widely studied for the last 60 years. In this talk, we will introduce similar norm-attainment concepts adapted to the space of nuclear operators  $\mathcal{N}(X, Y)$ , and also to the projective tensor product space  $X \widehat{\otimes}_\pi Y$ . We will exhibit positive and negative examples of whether or not norm-attainment holds in those cases. We will also study when such norm-attaining elements can be dense, and show that this is the case for many Banach spaces  $X$  and  $Y$  which, in particular, covers all classical Banach spaces (for instance,  $L_p$  spaces,  $L_1$  predual spaces, Banach spaces with a monotone Schauder basis, etc.). However, we also show that there exist spaces  $X$  and  $Y$  for which the norm-attaining elements in  $X \widehat{\otimes}_\pi Y$  are not a dense set.

The author was supported by the Spanish Ministerio de Universidades, grant FPU17/02023, and by project MTM2017-83262-C2-1-P/MCIN/AEI/10.13039/501100011033 (FEDER). Joint work with Sheldon Dantas, Mingu Jung and Abraham Rueda Zoca (see [1]).

### References

[1] Sheldon Dantas, Mingu Jung, Óscar Roldán and Abraham Rueda Zoca, *Norm-attaining tensors and nuclear operators*, Mediterranean Journal of Mathematics **19**(1) (2022), Paper no. 38, 27pp.

## Mean ergodic composition operators on spaces of holomorphic functions

*Daniel Santacreu*, Instituto Universitario de Matemática Pura y Aplicada IUMPA & Universitat Politècnica de València, Spain

We study mean ergodic composition operators on infinite dimensional spaces of holomorphic functions of different types when defined on the unit ball of a Banach or a Hilbert space: that of all holomorphic functions, that of holomorphic functions of bounded type and that of bounded holomorphic functions. Several examples in the different settings are given.

## Real Turrittin's Theorem and applications to trajectories of vector fields

*Fernando Sanz Sánchez*, Universidad de Valladolid, Spain

A classical result known as Turrittin's Theorem establishes normal forms for a system of linear ODEs with complex formal meromorphic coefficients. Essentially, the system can be diagonalized, up to a finite jet of degree equal to the Poincaré rank, by means of polynomial gauge transformations and ramifications. In this talk, we present a version of this result in the case of the real base field, where we require that all the transformations are given by matrices with real coefficients. This is a joint work with M. Barkatou and F. A. Carnicero.

In the second part of the talk, we present a result where real Turrittin's Theorem plays an important role. Namely, we show that to any given formal invariant curve  $\Gamma$  of a real analytic vector field  $\xi$  at  $(\mathbb{R}^n, 0)$  there corresponds an actual trajectory of  $\xi$  accumulating to 0 and asymptotic to  $\Gamma$ . This is a joint work with F. Cano and O. LeGal.

## On the maximal extension in the mixed ultradifferentiable weight sequence setting

*Gerhard Schindl*, University of Vienna, Austria

For the ultradifferentiable weight sequence setting it is known that the Borel map which assigns to each function the infinite jet of derivatives (at 0) is surjective onto the corresponding weighted sequence class if and only if the sequence is strongly nonquasianalytic for both the Roumieu- and Beurling-type classes. Sequences which are nonquasianalytic but not strongly nonquasianalytic admit a controlled loss of regularity.

We determine the maximal sequence for which such a mixed setting is possible and get information on the controlled loss of surjectivity in this situation (for both types).

Moreover, we compare the optimal sequences (solutions) for the appearing mixed strong nonquasianalyticity conditions.

Finally, we generalize these techniques and notions to the weight matrix setting, i.e. to one-parameter families of weight sequences. This approach allows the unified treatment of classes defined by weight sequences and weight functions and hence we can compare the optimal solutions in both settings. Here we focus on the Roumieu-type.

Joint work with David Nicolas Nenning and Armin Rainer.

## The real Gamma and Zeta functions are jointly o-minimal

*Patrick Speissegger, McMaster University, Canada*

We develop multisummability, in the positive real direction, for generalized power series with natural support, and we prove o-minimality of the expansion of the real field by all multisums of these series. This resulting structure expands (i) the expansion of the real field by all multisummable (in the positive real direction) power series, and (ii) the reduct of  $\mathbb{R}_{an^*}$  generated by all convergent generalized power series with natural support. In particular, its expansion by the exponential function defines both the Gamma function on  $(0, \infty)$  and the Zeta function on  $(1, \infty)$ .

## Factorization theorems in Denjoy-Carleman classes associated to representations of $(\mathbb{R}^d, +)$

*Jasson Vindas, Ghent University, Belgium*

The purpose of this talk is to discuss a strong factorization theorem of Dixmier-Malliavin type for ultradifferentiable vectors associated to broad classes of representations of  $(\mathbb{R}^d, +)$  on sequentially complete locally convex Hausdorff spaces. We employ our results to show that various convolution algebras and modules of ultradifferentiable functions commonly occurring in analysis satisfy the strong factorization property. In general, a module  $\mathcal{M}$  over a non-unital algebra  $\mathcal{A}$  is said to have the *strong* factorization property if  $\mathcal{M} = \{a \cdot m \mid a \in \mathcal{A}, m \in \mathcal{M}\}$ .

Collaborative work with Andreas Debrouwere and Bojan Prangoski.

## Continuously differentiable functions on compact sets

*Jochen Wengenroth*, Universität Trier, Germany

We consider the space  $C^1(K)$  of  $\mathbb{R}^m$ -valued continuously differentiable functions  $f : K \rightarrow \mathbb{R}^m$  on an arbitrary compact set  $K \subseteq \mathbb{R}^d$  defined by the usual affine-linear approximability, i.e., there is a continuous function  $f' : K \rightarrow L(\mathbb{R}^d, \mathbb{R}^m)$  with

$$\frac{\|f(x+h) - f(x) - f'(x)(h)\|_{\mathbb{R}^m}}{\|h\|_{\mathbb{R}^d}} \rightarrow 0 \text{ for all } x \in K$$

endowed with the norm

$$\|f\|_{C^1(K)} = \inf\{\sup\|f'(x)\|_{L(\mathbb{R}^d, \mathbb{R}^m)} : x \in K\} : f' \text{ is a continuous derivative of } f\}.$$

We characterize the completeness of this space and prove that the restriction space  $C^1(\mathbb{R}^d|K) = \{f|_K : f \in C^1(\mathbb{R}^d)\}$  is always dense in  $C^1(K)$ . The space  $C^1(K)$  is then compared with other spaces of differentiable functions on compact sets.

Joint work with Leonhard Frerick and Laurent Loosveldt.

# List of Participants

Vicente Asensio viaslo@doctor.upv.es	IUMPA Universitat Politècnica de València
José Bonet jbonet@mat.upv.es	IUMPA Universitat Politècnica de València
Sergio Alejandro Carrillo Torres sergio.carrillo@usa.edu.co	Universidad Sergio Arboleda
Céline Esser celine.esser@uliege.be	University of Liège
Carmen Fernández Rosell fernand@uv.es	Universitat de València
Antonio Galbis antonio.galbis@uv.es	Universitat de València
Jorge Galindo jgalindo@uji.es	Universitat Jaume I
Pablo Galindo galindo@uv.es	Universidad de Valencia
M. Carmen Gómez Collado mcgomez@mat.upv.es	IUMPA Universitat Politècnica de València
Michiel Huttener michiel.huttener@ugent.be	Ghent University
Javier Jiménez Garrido jimenezjj@unican.es	Universidad de Cantabria IMUVA
Enrique Jordá Mora ejorda@mat.upv.es	IUMPA Universitat Politècnica de València
David Jornet Casanova djornet@mat.upv.es	IUMPA Universitat Politècnica de València
Thomas Kalmes thomas.kalmes@math.tu-chemnitz.de	Chemnitz Technical University
Thomas Lamby Thomas.Lamby@uliege.be	University of Liège
Alberto Lastra Sedano alberto.lastra@uah.es	Universidad de Alcalá
María Martín Vega maria.martin.vega@uva.es	Universidad de Valladolid
Félix Martínez-Giménez fmartinez@mat.upv.es	IUMPA Universitat Politècnica de València



Claudio Mele claudio.mele1@unisalento.it	Università del Salento
Ignacio Miguel Cantero ignacio.miguel@uva.es	Universidad de Valladolid IMUVA
Arman Molla A.Molla@uliege.be	University of Liège
Jorge Mozo Fernández jorge.mozo@uva.es	Universidad de Valladolid
David Nicolas Nenning david.nicolas.nenning@univie.ac.at	University of Vienna
Lenny Neyt Lenny.Neyt@UGent.be	Ghent University
Samuel Nicolay S.Nicolay@uliege.be	University of Liège
Alessandro Oliaro alessandro.oliaro@unito.it	University of Torino
Alfred Peris aperis@upv.es	IUMPA Universitat Politècnica de València
Armin Rainer armin.rainer@univie.ac.at	University of Vienna
Francisco Rodenas frodernas @ mat.upv.es	IUMPA Universitat Politècnica de València
Alberto Rodríguez Arenas arenasa@uji.es	Universitat Jaume I
Óscar Roldán oscar.roldan@uv.es	Universitat de València
Daniel Santacreu Ferrà santacre@uji.es	IUMPA Universitat Politècnica de València
Fernando Sanz fsanz@agt.uva.es	Universidad de Valladolid
Javier Sanz javier.sanz.gil@uva.es	Universidad de Valladolid IMUVA
Gerhard Schindl gerhard.schindl@univie.ac.at	University of Vienna
Patrick Speissegger speisseg@math.mcmaster.ca	McMaster University
Jasson Vindas jasson.vindas@ugent.be	Ghent University
Jochen Wengenroth wengenroth@uni-trier.de	Universität Trier

# Useful Information

Talks will be held in **AULA 105** at the lecture room building (Aulario) close to the Science Faculty building of the University of Valladolid.  
Campus address: Paseo de Belén 7, Valladolid 47011 (Spain).

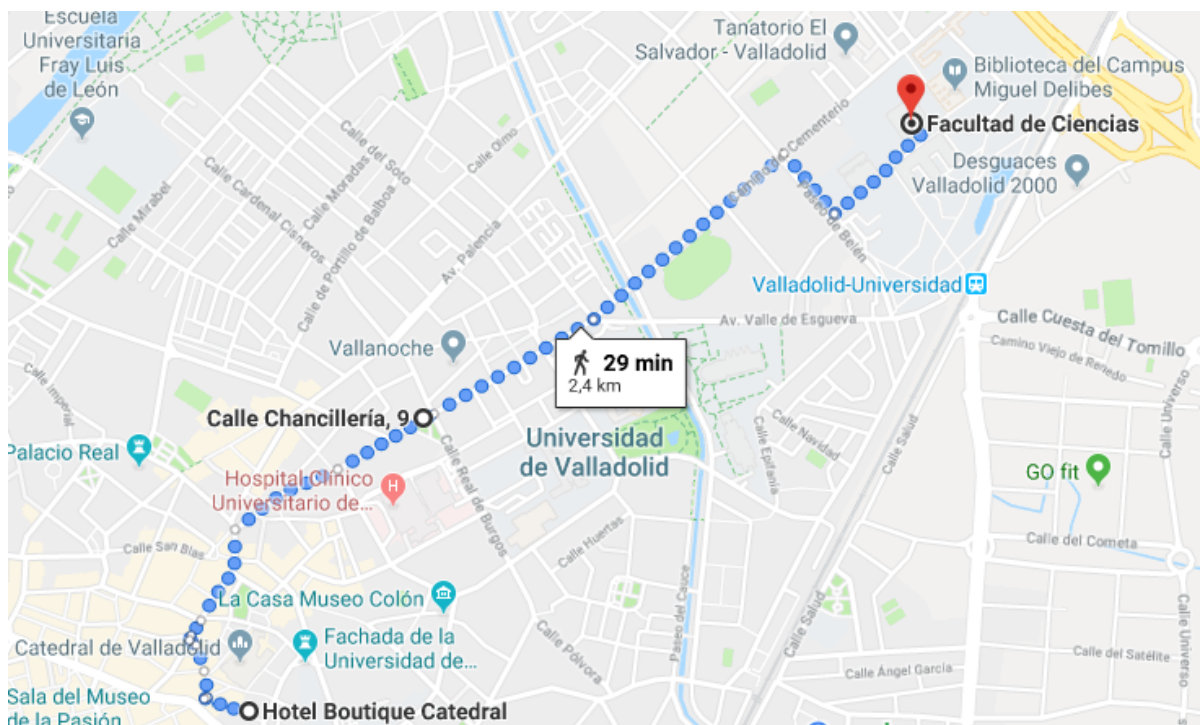
AULA 105 is equipped with a projector and blackboard.

Talks by the participants coming to Valladolid will be broadcasted using Webex, if they agree to do so.

Wi-Fi will be available during the conference. The University of Valladolid also provides access to an eduroam network.

## How to get to the Science Faculty?

**Walking:** (around 29 minute long) through the center of the City from the Hotel Boutique Catedral to the Science Faculty (or vice versa) can be found here:



**By Bus:** line 8, which runs every 12 minute. Every trip is 1,50 euro, which you should pay to the driver when getting on the bus through the front door (**only credit card is accepted, no cash is allowed**):

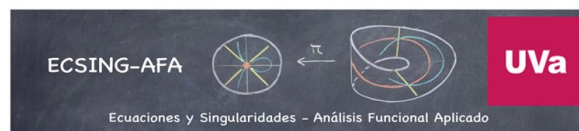
- **When going to the Faculty:** First walk from the hotel to the Plaza de la Fuente Dorada (3 minutes), and take there line 8 (the sign in the upper part of the vehicle front reads «8 Bº BELEN») to the Campus de Miguel Delibes (last stop), where the Science Faculty is located (a 2 minute walk from the stop).
- **When coming from the Faculty:** Take the bus in the stop near the Faculty, where only line 8 starts, and get off in Santuario Street, then walk to the hotel (around 3 minute walk).

# Partner Institutions and Sponsors

The following institutions, research groups and research projects are supporting this Workshop:



Research Project PID2019-105621GB-I00, Ministerio de Ciencia e Innovación, Gobierno de España



Research Groups (GIR) AFA and ECSING, University of Valladolid



Science Faculty, University of Valladolid    Instituto de Investigación en Matemáticas de la Universidad de Valladolid, IMUVA



Turist Office of the City Hall of Valladolid



